Kinematic variational principle for vortical structure of Euler flows and beyond

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Coherent vortices, being durable for some time, are often observed nature. These vortices may be modeled as steady solutions of the Euler equations for an inviscid incompressible fluid. A steady incompressible Euler flow is characterized as an extremal of the total kinetic energy with respect to perturbations constrained to an isovortical sheet (coadjoint orbits). Kelvin (1878) argued that an elongated vortex of circular cross-section is realized as a maximum energy state, and recently his conjecture has been numerically and mathematically confirmed. This guarantees its stability if the disturbances are restricted two-dimensional isovortical ones. An isovortical perturbation preserves vortex-line topology and is expressible most efficiently by the Lagrangian variables. I will show how topological ideas work in the variational formulation for characterizing a steady solution of the Euler equation [2, 3, 4].

This is generalized to a steadily moving vortical flow. According to Kelvin-Benjamin's principle [5], a steady distribution of vorticity, relative to a moving frame, is realized as the state that maximizes the total kinetic energy, under the constraint of constant hydrodynamic impulse, with respect to variations preserving the vorticity-field topology. Combined with an asymptotic solution of the Euler equations for a family of vortex rings, we can skip the detailed solution for the flow field to obtain the translation velocity of a vortex ring valid to third order in a small parameter, the ratio of the core radius to the ring radius [6].

A discussion is made of possible future directions of the research.

References

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