Anomalous dissipation as a trigger for the energy cascade in 3D inviscid flows

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Abstract

It was conjectured by Onsager in 1949 that "...in three dimensions a mechanism for complete dissipation of all kinetic energy, even without aid of the viscosity, is available." More precisely, Onsager conjectured that the minimal spatial regularity of a (weak) solution to the 3D Euler equations needed to conserve energy is $\frac{1}{3}^+$, and that in the case the energy is not conserved, the energy dissipation due to the lack of regularity – the anomalous dissipation – triggers the energy cascade that continues *ad infinitum* dissipating all the (kinetic) energy in the flow.

The purpose of this lecture is to show that the anomalous dissipation is indeed capable of triggering the energy cascade – in physical scales of the flow – which then continues indefinitely confirming Onsager's predictions. The main device in the proof is suitable ensemble averaging of the local energy inequality.