Non-“equilibrium” oscillations in two-dimensional Euler equations

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Two-dimensional (2D) flows of fluids with high Reynolds numbers, such as oceanic and atmospheric flows, often show large-scale patterns, including Jupiter’s red spot and Kuroshio of the north pacific ocean. Studies for the last two decades have revealed that such a large-scale stationary pattern is described as an *equilibrium* state of statistical mechanics, i.e., the state of maximizing a Shannon entropy functional with respect to the vorticity level of the 2D Euler equations [2].

Here, let us recall ordinary thermodynamic systems, to which equilibrium statistical mechanics is most typically applied. When the system is far from equilibrium, it often shows a non-equilibrium temporal motion on macroscopic scales, such as limit cycle, through bifurcations.

By thinking parallelly, when the above 2D fluid is far from the reference stationary pattern, we expect a large-scale non-stationary motion, through bifurcations. In the present talk, we indeed discover temporal oscillation in the 2D Euler equations [1].

We consider the 2D Euler equations on a doubly periodic domain. When the initial condition is near the entropy maximizing stationary flow, the system relaxes close to this stationary flow. When the initial condition is very far from the reference stationary flow, in contrast, we observe a non-stationary pattern of flow; positive and negative coherent vortices moves pairwise along stream line. This motion lasts stably, without relaxing to the reference stationary state, and therefore regarded as a limit cycle on the largest scale.

Investigating this phase transition from stationary to non-stationary flow, by introducing an order parameter, shows that this oscillation appears through Hopf bifurcation. This indicates the structure of low-dimensional dissipative dynamical system embedded in the high-dimensional conservative dynamics of the 2D Euler equations.

In addition, we examine how the large-scale oscillation is kept stable, through a dynamic self-consistent theory. We explain that this oscillation is sustained by collectively organizing a self-oscillating state. Note that this mechanism is common to collective oscillation in N-body Hamiltonian systems [3].

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[3] H. Morita and K. Kaneko, Phys. Rev. Lett. **96**, 050602 (2006).

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