Residual vanishing of concentration arising in the mean field equations

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In this talk, we study the mean field equation on a two-dimensional compact Riemannian manifold (M, g) without boundary:

$$\begin{cases} -\Delta_g v = \lambda_+ \left(\frac{e^v}{\int_M e^v dv_g} - \frac{1}{|M|} \right) - \lambda_- \left(\frac{e^{-v}}{\int_M e^{-v} dv_g} - \frac{1}{|M|} \right) & \text{in } M \\ \int_M v dv_g = 0, \end{cases} \tag{$\sharp$}$$

where Δ_g, dv_g, $|M|$ and $\lambda_\pm$ are the Laplace-Bertrami operator, the volume form, the volume of M and non-negative constants, respectively. Let $\{\lambda_{\pm,n}\}$ be sequences of non-negative constants satisfying $\lambda_{\pm,n} \to \lambda_\pm$ for some non-negative constants $\lambda_\pm$, and let $\{v_n\}$ be a solution sequence to ($\sharp$) corresponding to $\{\lambda_{\pm,n}\}$. Then, the following alternative holds (by passing to a subsequence): (i) compactness (ii) one-sided concentration (iii) concentration, see [OS06] for details. We focus on (iii) in this talk, and assume that it holds. Then, $\mathcal{S}_\pm \neq \emptyset$, $\mathcal{S}_\pm$ are finite, and there exist non-negative functions $r_\pm \in L^1(M) \cap L^\infty_{loc}(M \setminus \mathcal{S}_\pm)$ such that

$$\lambda_\pm \frac{e^{\pm v}}{\int_M e^{\pm v} dv_g} =: \mu_\pm \overset{*}{\rightharpoonup} r_\pm dx + \sum_{x_0 \in \mathcal{S}_\pm} m_\pm(x_0) \delta_{x_0} \quad \text{in } \mathcal{M}(M) \tag{1}$$

with $m_\pm(x_0) \geq 4\pi$ for all $x_0 \in \mathcal{S}_\pm$, where

$$\mathcal{S}_\pm = \{x \in M \mid \exists x_n \to x \text{ s.t. } v_n(x_n) \to \pm\infty\}.$$

Here, the term "residual vanishing" means

Proposition 1. $r_\pm = 0$ *in (1).*

By the proposition, we can refine the result of [OS06] stated above. We also study the Sawada-Suzuki model ([SS08]) which can be regarded as a more general model of ($\sharp$). The blowup analysis for this model is done in [ORS10]. We see that the similar property (i.e., residual vanishing) holds for their result.

[OS06] H. Ohtsuka and T. Suzuki, Adv. Differential Equations **11** (2006) 281-304.

[ORS10] H. Ohtsuka, T. Ricciardi, and T.Suzuki, J. Differential Equations **249** (2010) 1436-1465.

[SS08] K. Sawada and T. Suzuki, Theoret. Appl. Mech. Japan **56** (2008) 285-290.